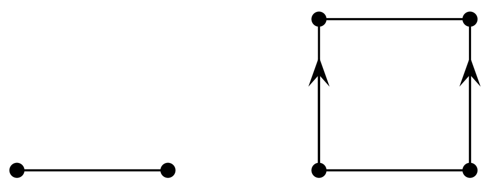
FOURTH DIMENSION WRITTEN SUPPLEMENT

As we saw in class the fourth dimension comes in two ‘flavors,’ that of time and that of space. This supplement will focus on the fourth dimension of space. This can be a hard concept for our minds to comprehend because we only experience 3 spatial dimensions or directions: up/down, left/right, forward/backward. Any other directions we experience are just combinations of these. By its very nature (and name!) the *fourth* dimension is something that is beyond the *three* dimensions with which we are surrounded all our lives.

Even though we don’t experience a fourth dimension of space, we can learn about it and explore it by using mathematics and logical reasoning. For the dimensions with which we are familiar there is a simple way to build up from one to the next, and we can use this method to begin to think about the fourth dimension. To build a zero-dimensional, 0D, point into a one-dimensional, 1D, segment, we push the point a distance away from itself and trace out a segment:

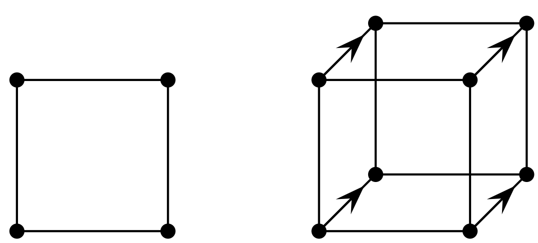
OD 1D

Thus, through movement and tracing we have created a one-dimensional object from a zero-dimensional object. Continuing, we push the segment in a direction perpendicular to itself and trace out that movement to create a two-dimensional square:



1D 2D

Continuing in this way we can move from two dimensions to three dimensions. This time we push the square in a direction perpendicular to itself and trace out its path:



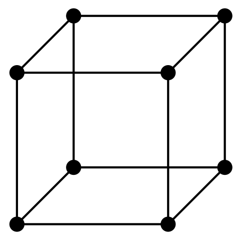
2D 3D

Our eyes are trained to see the image on the right above as a cube, but, of course, it is really not a cube. It is a flat drawing on a flat piece of paper (or on a flat computer screen). To create a real cube out of a square we would have to push it up out of the screen, but we cannot do that so we represent that direction using diagonal lines. It’s something we’ve seen enough that we’ve come to accept it.

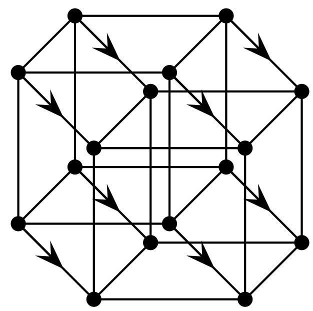
So in going from a line segment to a square we went in a perpendicular direction (at 90^o angles) to what was there, and in going from a square to a cube we went in a perpendicular direction to what we had already (even though the lines we used to trace look diagonal rather than perpendicular, we know the image is supposed to pop off the page, perpendicular to the other lines). We’re up to three dimensions. In order to create a four-dimensional cube, also known as a hypercube, we need to push our cube in a direction perpendicular to all the others we have used so far.

Where is that direction?

It is a direction we cannot see or physically find, just as a 2-dimensional creature living in a world that is a flat plane would have no idea where the third dimension is and would not be able to physically create a cube. However, we can use the process of tracing we've been using all along in order to create a diagram of four-dimensional cube. To do so, we 'push' our cube in a direction perpendicular to the others; though we cannot physically do this we can represent it with diagonal lines as we did in going from square to cube.



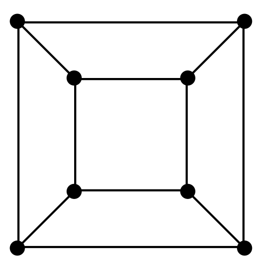
3D



4D

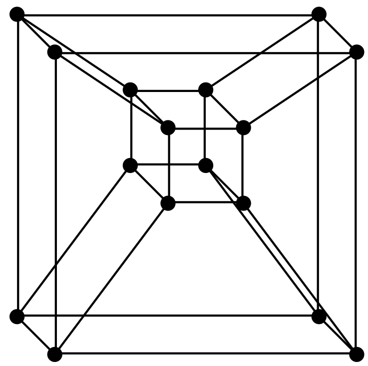
The cube is already a bit of a stretch, but since we are familiar with cubes in real life we can immediately recognize the representation of one on a flat surface. The 4D cube, or hypercube, on the other hand, is not something we are familiar with, so, though this drawing helps a bit in getting the idea, we're pretty far removed from anything we can truly visualize.

There are a couple of other ways of approaching this that also involve building up from a cube. Another way to represent a cube is to imagine looking into it as if it were a box:



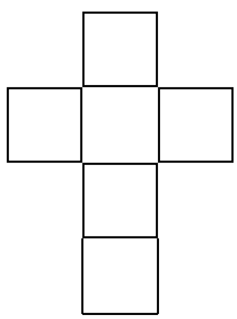
You know from your experience with cubes and boxes that the square that seems to be in the middle (or on the 'inside') here isn't really on the inside, nor is it really smaller than the other square. It's just a matter of perspective. If this cube is a box you are looking into, then the bottom is farther away than the top, and therefore appears smaller. Also, you know that a cube has 6 square sides, but this shape appears to only contain two squares. The four other shapes seem to be trapezoids with slanted sides but this is an illusion due to perspective as well. All of the closed figures in this cube actually represent perfect squares.

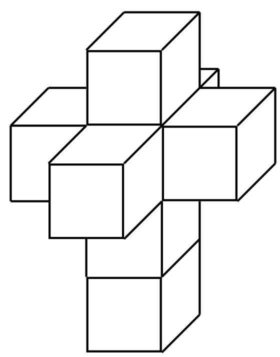
We can take a similar approach to a hypercube. Here is the result:



Here too it looks like we have a smaller shape inside a larger shape and as if we have slanted shapes around the inside, but this is not the case. The fact that one cube looks smaller is again a matter of perspective. It is simply further away. The shapes with slanted lines around the 'inside' are actually cubes. This image is of the same 4D shape as you see at the top of the page but just from a different perspective.

Here is one other option in terms of thinking about the construction of a hypercube based on the construction of a cube. If we wanted to make a paper cube by cutting a model out of a piece of paper and taping sides together, we could use the image on the left in order to accomplish this:





If we wanted to make the model cube, we would just cut out the figure on the left, and fold the sides 'up' off the flat plane of the paper and tape them together. This makes perfect sense to us, but to a creature living in 2 dimensions it would be impossible to do and even pretty hard to imagine because he would have no concept of the direction (or dimension) 'up' off of the plane. In terms of making a genuine model of a 4-dimensional cube, we find ourselves in the same situation as the flatland creature. We could make our hypercube IF we could figure out where the direction is in which we have to fold it! But we live in the 3^{rd} dimension, so we don't have enough space in our space, so to speak.

1) Although we cannot "fit" a 4-dimensional (or 5-dimensional) object into our 3-dimensional world, we can use logic and mathematical patterns to discover what higher dimensional objects are like. Just as a cube is a 3-dimensional analog of a square, a hypercube is a 4-dimensional analog of a cube, and a hyper-hypercube is a 5-dimensional analog of a cube. Use logic based on the lecture, the video, and these materials, and use the pattern below to determine how many vertices (i.e. corners), edges (i.e. line segments), faces, cubes and hypercubes are in each of the objects whose rows have been left blank for you to fill in.

2) Time is sometimes looked at as a fourth dimension. Dimensions are sometimes called 'degrees of freedom.' We do have freedom of movement in our 3 spatial dimensions: left/right, forward/back, up/down. If time is a dimension like these others it seems like we should be able to move freely in time as well – that is, it seems we should be able to travel in time. For this problem answer parts (a) and (b) on the next page.

	# of vertices	# of edges	# of faces	# of cubes	# of hypercubes	# of hyper-hypercubes
point	1	0	0	0	0	0
segment	2	1	0	0	0	0
square	4	4	1	0	0	0
cube	8	12	6	1	0	0
hypercube						
hyper-hypercube						

(a) Do you think time travel will ever be possible? Why or why not?

(b) Imagine the possibilities of time travel. If you were given one opportunity to travel in time, would you travel to the past or the future? Why? What would you do?