Math 32/101 Name_____

Infinity Worksheet

In class we showed that the set of natural number and the set of even numbers are equivalent, even though it seems that there should be half as many even numbers. Being equivalent means they have the same cardinality and that they have the same number of elements. While showing this, we accomplished something else as a bonus; we proved that the natural numbers are infinite . . . not that that was actually in doubt, but being able to match the elements of a set with a proper subset of itself is the very definition of an infinite mathematical set.

1) The square numbers are a proper subset of the natural numbers. WHY?

2) Find a rule matching the natural numbers and the square numbers (in both directions)

$\mathfrak{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

$\mathfrak{S} = \{1, 4, 9, 16, 25, 36, \ldots\}$

3) You have proven two things here. What are those two things?

4) Find an infinitely long proper subset of the whole numbers and list is below.

5) Why is your list above a proper subset of the whole numbers?

6) Find a way to match the whole numbers to their proper subset that you've chosen.

$\mathbb{W} = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$

7) By making this matching you have proven, according the Cantor's Definition, that _____

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8) In class we found that most infinitely long sets of numbers that we are familiar with have the same cardinality – in other words they are the same level of infinity. What is the symbol for this size of infinity and what is its name?

9) In class we also found that one of the infinite sets of numbers that we are familiar with actually constitutes a higher level of infinity. Which set was that? What symbol represents that size of infinity, and why is that symbol used?