Math 101 - Section __________ **NAME___________________________**

PROBLEM SOLVING

COMMENTS & DIRECTIONS: The items contained here are significant "problems!" They are not mere "exercises." We will begin them on the day you receive this handout, you should be thinking about them and working on them for two to three weeks at which point we will revisit them as a class when we pick up the "Problem Solving" theme again. Do not wait to get started, as they are related to number theory, which we have recently been working on, and because you need time in which to try to wrap your mind around them a bit before we tackle them together! You will be asked problems like this on test three and on the final exam, so it is very important you work at them on your own in order to develop your problem-solving skills before we work them together in class. You'll have a long period of time on which to work on these, but don't put it off!

1) Go to the following link and try to figure out how the trick works. Explain in detail. (If the link is broken, do a Google search for 'crystal ball mind reading trick.')

http://www.readthemind.com

NOTE TO TUTORS: This is a long-term PROBLEM-SOLVING assignment. Feel free to point students in the right direction, but do not give full steps or answers. The key is to help students THINK about the problem. THANK YOU!

2) Do the "Phone Number Problem" below and answer the questions that appear after the problem.

Phone Number Problem

1) Type into your calculator the first 3 digits of your phone number (not area code).
2) Multiply by 80 (and hit the = sign).
3) Add 1 (and hit the = sign).
4) Multiply by 250 (and hit the = sign).
5) Add the last 4 digits of your phone number (and hit the = sign).
6) Repeat step 5.
7) Subtract 250 (and hit the = sign).
8) Divide by 2 (and hit the = sign).

QUESTIONS: What is the result? Are you surprised? **HOW** does this trick work? Write out your answers to these questions in detail.

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3) One million light bulbs are controlled by one million switches numbered in order from 1 to 1,000,000. All switches are in the off position to begin. Starting at 1, every switch is flipped. Next, starting at 2, every second switch is flipped. Then starting at 3, every third switch is flipped. (Of course, if a switch was off, flipping it turns it on and vice versa). This continues until the millionth switch is reached. After all this, ***how many*** light bulbs will be on?

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4) Most or all of us probably remember some letter or number games from childhood. Perhaps you remember games from long car trips like finding all the letters of the alphabet on signs or license plates or games you might hear on the playground like "eenie-meenie-minie-moe" or "I one it, I two it . . . I jumped over it and you ate it!" One such counting game has the following rules:

> This is a two person game, and the winner is the person who says "21." We start with the number 1, and each of us can count one or two or three numbers at a time."

Just like with "eenie-meenie-minie-moe" there is a way to win every time if you're clever and set it up just right. Play this game a few times (you might want to keep a record of which numbers each player says), think about strategy as you play, and then answer the following questions:

1. What strategy will allow you to win every time?
2. How can you win this game in general even if you are counting to a number other than 21 or can count by groups of more than three numbers?

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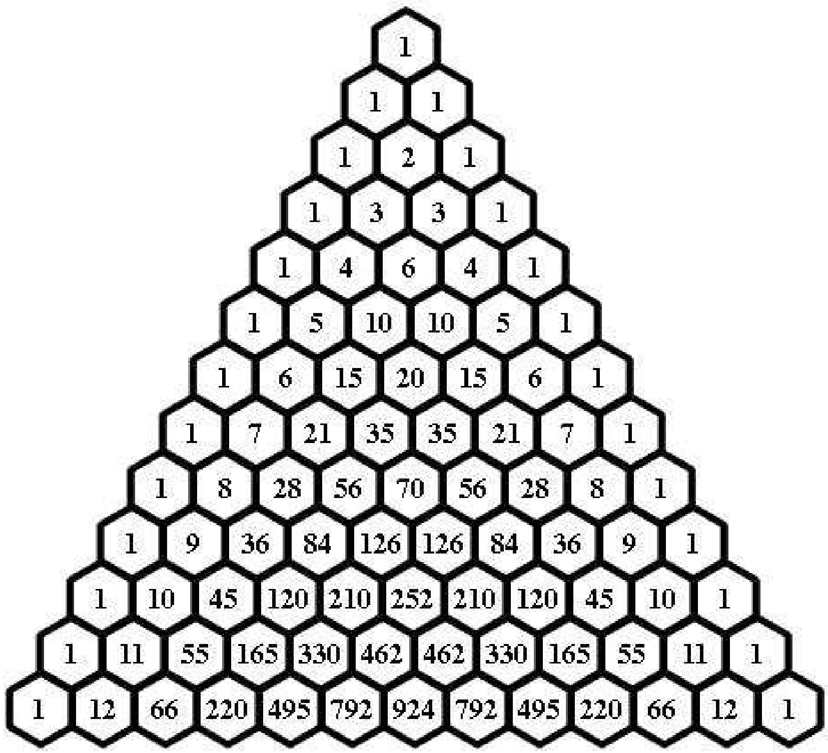
5) There are hundreds of patterns in the following triangle. Find as many patterns in it as you can (including how it is created!). Patterns may be along the rows, columns or diagonals or in different configurations. I have give you a few copies of this over the next few pages so you can try finding patterns without having to erase. You may list more than one pattern per page if you would like. At the bottom of this page I have listed a couple of the patterns and have asked some questions to get you started finding your own.



Here are a few patterns and questions to get you started:

- This triangle has horizontal symmetry.
- There are ones down the left and right sides of the triangle.
- The counting numbers can be found in the second diagonal in.
- If you choose a number in the interior, like the 4 in the fifth row down, and you look at the numbers circling it, the products of alternating numbers are equal - that is: $3 \times 10 \times 1 = 6 \times 5 \times 1$. This works for the numbers surrounding any cell in the interior.

Here are some questions that might help you in your search:

- Is there anything special about any of the other diagonals?
- What happens if you add up all the numbers in a row and look at the sums of each row?
- Can you find a pattern made up of odd numbers?
- Can you find the Fibonacci Numbers in the triangle?
- What are the first few powers of 11? Can you find them in the triangle? Do they continue all the way?

Those are just ideas to get you started. There are many, MANY patterns that can be found in this triangle! See how creative you can be in your search!

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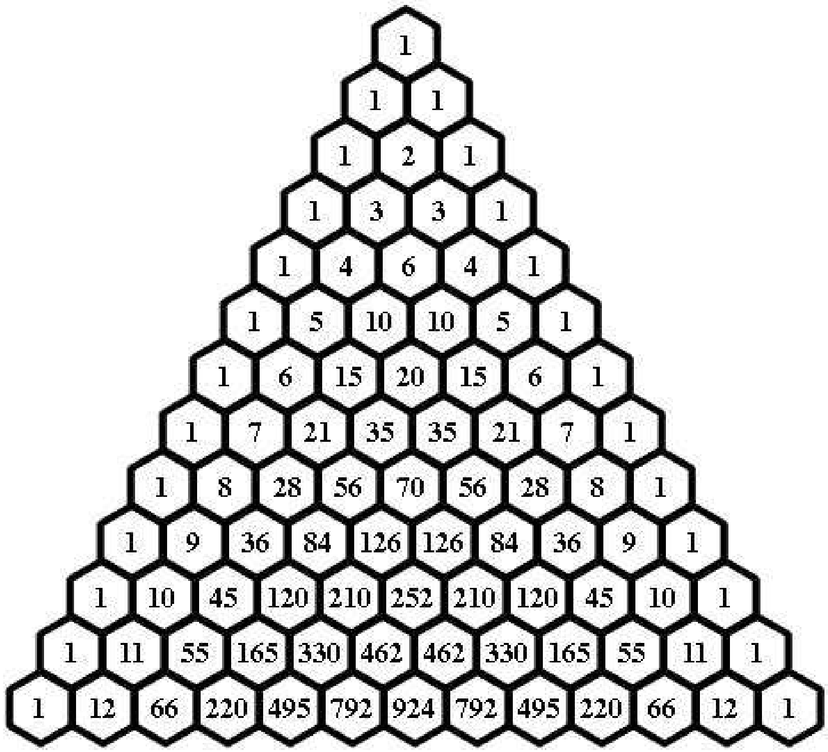

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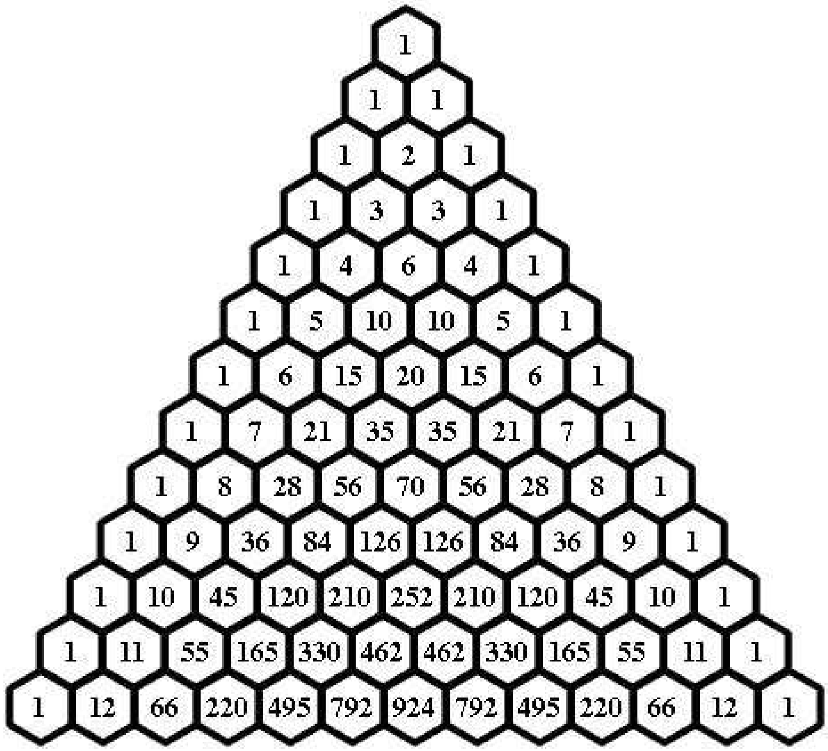

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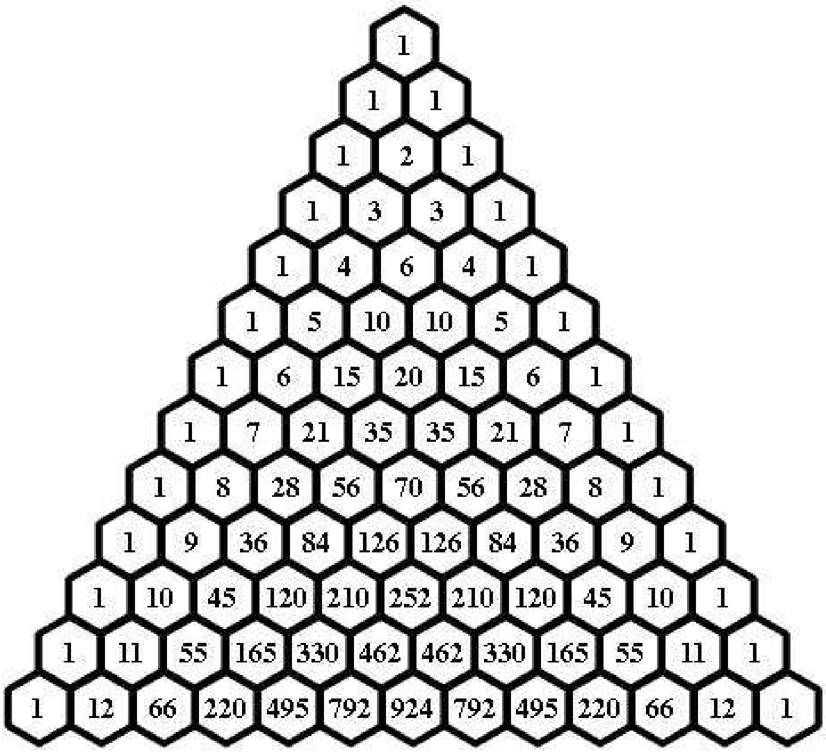

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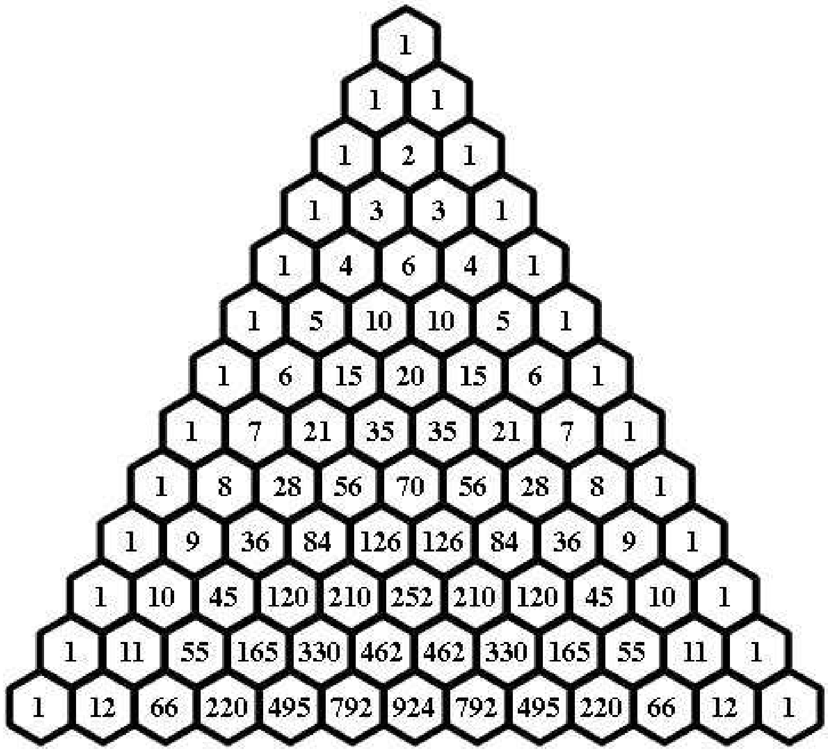

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