MATH 101 **Name:** ______________________________

FINAL EXAM B **Spring 2012**

DIRECTIONS: This is an OPEN NOTE exam. You may also use a standard calculator (but not a cell phone). If it is possible to show work on a problem then ***be sure to show your work***. You may earn no credit if work is not shown. This is especially necessary since you get to use your notes and a calculator; you need to give me something to show me you actually understand.

1. Fill in the blank with the mathematician described. Each answer is worth three points.

________________________ The greatest achievement of this mathematician and scientist was his discovery of the mathematical equations governing electromagnetism. He was born in Scotland, and, like so many of the mathematicians we studied, he showed tremendous mental ability at an early age. By age 8 he had memorized the longest Psalm in the Bible, Psalm 119, which he may have done because his mother passed away that year. He was also published by age 14. Einstein thought so highly of him that he kept a photo of this mathematician in his office alongside pictures of Faraday and Newton.

________________________ As we saw repeatedly in the presentations women living before the 20^{th} century had tremendous difficulties to overcome if they wanted to do mathematics. This woman was no exception. Her parents noticed her interest in mathematics and forbade her to study it. She began to sneak down to her father's library at night to study, but when her parents found out they removed clothing and candles from her bedroom so that she would not be able to leave her room. She just wrapped herself up in a blanket and kept studying anyway. The reason she was this driven to study mathematics is that she had read the biography of Archimedes and knew how he died at the hands of a Roman soldier rather than stop doing his math, and she thought that math must be pretty amazing to draw someone in that deeply. Eventually she began corresponding with mathematicians by hiding behind the pseudonym M. Le Blanc.

________________________ The movie *A Beautiful Mind* is a fictionalized account of the life of this mathematician. He earned his Ph.D. at Princeton but rarely attended class because he was arrogant and felt that going to class would destroy his creativity. He later taught at MIT where the students called him "The Kid Mathematician" because of his youthful looks. He began to struggle with schizophrenia and turned down a position he was offered, saying he was about to become ruler of Antarctica. He won the Nobel Prize in Economics for his Equilibrium Theory. After a long struggle, his mental health finally stabilized, and he is currently at Princeton as an emeritus professor.

__________________________ At a conference at the beginning of the 20^{th} century this mathematician presented 23 problems that he felt were the most important to mathematics. He taught at Göttingen and often bicycled or skied to work. In 1930 he was forced to retire because he was a Jew. Later he was asked about the mathematicians in Göttingen,and he said, “Mathematicians in Göttingen? There really are no mathematicians in Göttingen anymore.

__________________________ This man taught at Christ Church, Oxford. In mathematics he was mainly know for his work on logic, determinants, and voting theory, but you probably know him better as the author of the ‘Alice’ books. He was also an avid photographer, one of two best in England during Victorian times.

__________________________ This mathematician lived in the Castle of Merchiston in Scotland. Mathematically he is best known as being one of the inventors of logarithms and also for a basic calculator made up of ‘rods’ or ‘bones.’ He was deeply religious and felt his most important work was his theological writing about the biblical book of Revelation. Strangely, he also dabbled in necromancy and alchemy. He was very clever and found tricky ways to do such things as determine which of his servants was a thief and also how to keep the neighbor’s pigeons out of his crops; because his methods were so surprising and subtle he was thought to have magical powers.

__________________________ As with many of our mathematicians, this man was able to learn other languages easily. He invented Icosian Calculus, but he is best known for his work on quaternions. He had a breakthrough mathematical idea on a walk with his wife one day and was so excited that he took out his pen-knife and carved the solution onto the side of the bridge they were walking across. The solution he’d been looking for was $i^2 = j^2 = k^2 = ijk = -1$. Quaternions are used today in computer graphics particularly for rotations and orientation of images.

__________________________ This woman was the oldest of 21 children. Much of her mathematical interest and ability developed because she needed to tutor all her younger siblings! She is the first woman in the ‘western world’ to be known as a mathematician - although her wish was to be a nun! Her father, a professor, wouldn’t let her enter a convent, so she dedicated her life to math. She wrote a 1000-page book gathering all the math of her time. She spent the end of her life in charitable works, selling her possessions and giving money to the poor.

__________________________ This professor who is currently at Oxford has also taught at Princeton. He came across a theorem when he was 10 years old, and it so captured his imagination that he continued as an adult to pursue his dream of solving it. The theorem was known as Fermat’s Last, and it had baffled mathematicians for more than 300 years. He is married and has 3 daughters.

__________________________ This was the earliest woman mathematician that we heard about. She taught at the famous library of Alexandria where her father was librarian. She did a lot of original work, but is probably best know for her commentaries on the mathematical work of others, especially a commentary on the work of Diophantus. Sadly, it seems, she was not appreciated by everyone in her day. She was considered a pagan and was killed by an angry mob who scraped her skin off with shells, dragged her through the streets and burned her.

__________________________ He was a vegetarian because he believed everything had a spirit He believed the soul never died but cycled until it was finally freed. He founded and led a secret society that bore his name. This society or brotherhood was both a school and a religion. Those attending lived at the school, had no possessions and had to be vegetarians. He believed numbers could unlock the secrets of the universe. His favorite number was 1, because he believed it to be the origin of everything, but another number that was dear to his heart and to his followers was the number 10, which they called the 'tetractys' and considered to be perfect. He is probably most famous for the theorem that bears his name: $a^2 + b^2 = c^2$.

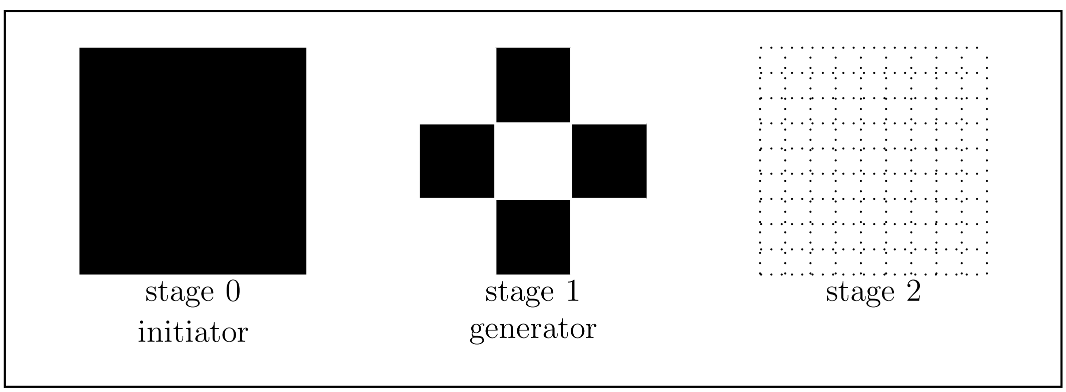
__________________________ He was a quintessential 'Renaissance Man' who tremendously impacted philosophy and mathematics. The city in France that he was born in is now named for him. As a young student in boarding school he got a 'free-pass' and was allowed, due to his poor health, to sleep in. This is when he did a lot of his thinking. He traveled abroad and spent time in the army, but he didn't take a position in the army because he wanted to reflect on things that bothered him. Philosophically he deconstructed his world and is well known for the phrase *Cogito ergo sum* ("I think therefore I am."). In mathematics he is best known for something we probably take for granted but which was actually such a turning point that such things as calculus would not have been possible without it. Unfortunately he went to work for Queen Christina of Sweden who insisted on having lessons at 5am in a drafty room in the dead of winter; this is most likely what led to his death from an inflammation of the lungs.

__________________________ One of the greatest scientists and mathematicians who ever lived - his father died 3 months before he was born, and his mother abandoned him, leaving him with his grandparents, so she could remarry. This had a strong impact on his life and left him feeling insecure. He was eventually reunited with his mother who took him out of school and wanted him to become a farmer. Eventually he did go to Cambridge, and he taught himself mathematics. During his college days the school had to shut down because of the plague; he went home and did a lot of his most important work at this time. He is one of the inventors of the calculus, and in physics we study his three laws of motion. He had strong religious views, did a lot of theological writing later in life. He was master of the mint in London for a time, and he was knighted by Queen Anne.

__________________________ Known as the Father of Geometry, he probably attended Plato's Academy; he taught at the famous library of Alexandria and was the founder of the mathematical school there. His book, *The Elements*, is one of the most widely studied books in history. It was one of the 3 most influential books in the life of Abraham Lincoln! One item in his writing has gotten a great deal of attention, and that is something known as the 'parallel postulate.'

__________________________ This is the guy whose name I spent all semester trying to get you to pronounce correctly - beginning with the sound "oy" - REALLY! His father was a pastor who wanted him to become a pastor as well, but Johann Bernoulli discovered what great mathematical ability he had and persuaded his father to allow him to pursue mathematics. He had a phenomenal, perhaps photographic, memory, which served him well later in life when he lost the sight in one eye and then the other. After losing the sight in one eye some of his friends began to call him 'Cyclops.' Among his many mathematical contributions was inventing and standardizing much of the notation we use today, wuch as $f(x)$, e, Σ, i, π, etc. He is also know for his work on the Königsberg Bridge Probem.

2. Given the initiator (stage 0) and generator (stage 1), draw stage 2 of the fractal. A grid has been provided to help you. (10 points)



3. Some questions about fractals: (15 points)

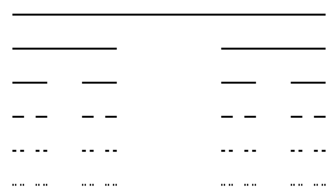
a) Find the self-similarity dimension of the fractal pictured in problem 2.
(Remember to show work.)

b) List 2 characteristics (out of the five) that all fractals have in common.

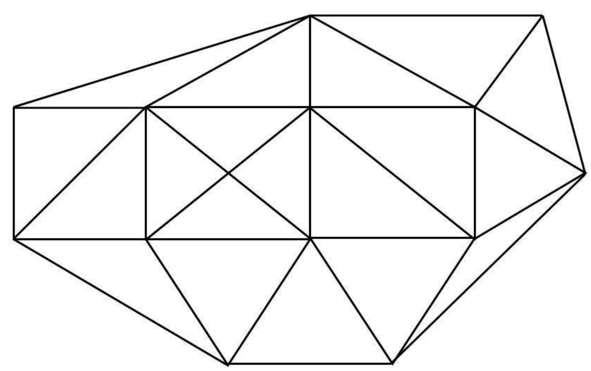
1) ______________________________________

2) ______________________________________

c) The name of the fractal pictured below is ______________________



4. Given the following graph, answer the questions below. You need to answer part 'a,' and then depending on your answer to 'a,' answer EITHER 'b' or 'c.' (15 points)



a) Is the graph above traversable? yes / no (Circle your answer.)

b) IF this one is traversable, does it matter where you start? yes / no

(If your answer is 'yes,' then circle a point where you could start.)

c) IF this one is not traversable then explain in the space below WHY it is NOT?

5. In each box below draw a shape that is representative of the dimension given. (10 points)

0-dimensional   1-dimensional   2-dimensional

3-dimensional   4-dimensional

6. One topic we worked with was polygons and tessellations. Together we developed formulas for angle measures of polygons. Consider the formulas below, and answer the questions associated with them. (10 points)

$$(n-2)180^o$$

a) The formula above is used to find ______________________________.

$$\frac{(n-2)180^o}{n}$$

b) The formula above is used to find ______________________________.

c) We use $n-2$ because ______________________________.

d) In order for a polygon to tessellate its angle measure has to divide into ______ evenly because ______________________________.

7. Some questions about dimensionality: (15 points)

a) Normally (as you saw in question 5) we only deal with dimensions that are whole numbers. What is one meaning that dimension of this type has?

b) Fractals are shapes that are complex in ways that defy standard defintions of dimension, so we developed together in class a new formula for finding dimension. When we use it to find fractal dimension we often get answers that are not whole numbers. What does fractal dimension tell us?

c) In the video we watched on the fourth dimension, Dr. Ed Burger talked about four amazing feats you can do in the fourth dimension that are impossible in just three dimensions. What was **one** of these things that he mentioned?

8. Answer the questions regarding networks below. (10 points)

a) Draw the shortest network between the 4 nodes below.

b) We considered two items in the natural world in which shortest networks show up. In other words, nature figured this out automatically before humans did mathematically! What were those two items?

9. LOGIC: We saw De Morgan's Laws in both our Set Theory Unit and in our Logic Unit. In logic, one form of De Morgan's Laws says that $\sim (\sim p \vee q) \equiv (p \wedge \sim q)$. Fill in the truth table below to show this equivalence, and give a one-sentence explanation below the chart explaining why the chart shows equivalence. (15 points)

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim (\sim p \vee q)$	$p \wedge \sim q$

10. SET THEORY: Write the name of the symbol in the blank behind the symbol.

(10 points)

$\cup$ stands for ____________________________

$\cap$ stands for ____________________________

$\in$ stands for ____________________________

$\subset$ stands for ____________________________

$\subseteq$ stands for ____________________________

$\mathbb{Q}$ stands for ____________________________

$\mathbb{Z}$ stands for ____________________________

$\mathbb{N}$ stands for ____________________________

$\mathbb{W}$ stands for ____________________________

$\mathbb{R}$ stands for ____________________________

11. Use the chart below to count from one to twenty in base 4. (10 points)

one	two	three	four	five
six	seven	eight	nine	ten
eleven	twelve	thirteen	fourteen	fifteen
sixteen	seventeen	eighteen	nineteen	twenty

12. Answer the following questions regarding bases: (10 points)

a) How is a number written in its own base? That is, how is *four* written in *base four* and how is *two* written in *base two* and how is *five* written in *base five*? HINT: Your answer should just be a number.

b) For systems that have a base larger than ten, what do you use to represent numbers such as 10 and 11 and 12 as single numerals?

13. Determine which of the labels below describes the number 1081: (15 points)

PRIME MERSENNE PRIME COMPOSITE DEFICIENT

PERFECT ABUNDANT

To earn full credit, circle the correct labels above and show your work on this problem below.

14. We considered some mathematical "magic tricks," and we looked at how they work - the telephone problem, the chocolate math problem, etc. Here is on such trick. Do the trick, answer the question asked in the trick, and then explain how the trick works using algebraic expressions and some dialog. (10 points)

Think of a number
Add 6
Double the result
Subtract 20
Subtract your original number
Add 8

Try the magic trick with three different numbers. What do you notice? How does it work?