MATH 101   Name: _____

TEST 3 (Ch. 2, Sec. 1.1 and Problem Solving)   Spring 2012

1)   We were reading a devotional book out loud together as a family on a recent evening, and the topic of following directions came up. The example the authors of the book used to illustrate this was a math problem! They asked the readers to do the following:

> "Choose a number, any number. Double it. Add 12. Divide by two.
> Subtract your original number. Now add one."

All of us followed these directions in our heads, and we all came up with the same answer even though we had started with different numbers to begin with. What answer did we all get? How does this work? (Use mathematical statements and English sentences to explain what is going on, and be sure to answer both questions I've asked.)

2)   Simplify each of the following factorial problems completely:
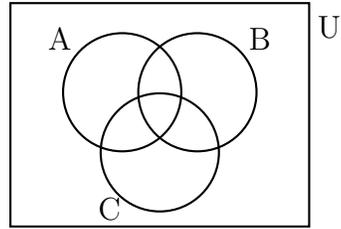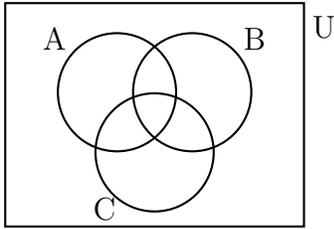
$$\frac{101!}{99!}$$ $$\frac{(x)!}{(x-1)!}$$

3) Shade a 3-loop Venn Diagram to show each of the following:

(a) $C \cap (A \cup B)'$ (b) $(B \cap C) \cup A$

4.   How many different license plates can be made in the format given below?

5.   State Cantor's definition of an infinite set:

6.   a) As we began our set theory unit in class I started out by asking you to compare the set of natural numbers, $\mathbb{N}$, and the even numbers, $\mathbb{E}$. We found this to be a mind-bending question because it seems that since both sets are infinite they must be the same 'size,' but, on the other hand, since we get the set of evens by removing half of the natural numbers (the odds) it seems there should be half as many evens as naturals. Which of these conclusions did we find to be true? (Check the correct answer below and then support your answer.)

____ The set of naturals and the set of evens have exactly the same number of elements.

____ The set of even numbers has half as many elements as the set of natural numbers.

Use the space here to 'prove' that what you chose above is correct:

b) Depending on your answer to part a, check the appropriate blank below and, if appropriate, fill in the blank.

____ The sets above are the same size, but another set that we studied in class, the set of _____ numbers is larger than the evens or naturals.

____ As I stated above $\mathbb{N}$ and $\mathbb{E}$ are of different 'sizes.' In fact, $n(\mathbb{N}) > n(\mathbb{E})$.

7.   There are 40 people at a meeting. If they all shake each other's hand exactly once how many handshakes are there? (As with all problems, be sure to show your work.)

8.   **Use** Cantor's definition to PROVE that $\{a, b\}$ is a finite set. (You will not earn credit for any reasoning used other than Cantor's definition.)

b) List the first four numbers of the set $\mathbb{S}$: $\mathbb{S} = \{x | x = n^2 - 1 \text{ where } n \in \mathbb{N}\}$

c) Represent the set of positive odd numbers (that is $\{1, 3, 5, 7, \ldots\}$) using set-builder notation.

9.   Use the following sets to answer parts a and b.

$\mathbb{U} = \{x | x \text{ is the name of one of the months in the year}\}$

$\mathbb{J} = \{x | x \in \mathbb{U} \text{ and } x \text{ begins with the letter J}\}$

$\mathbb{Y} = \{x | x \in \mathbb{U} \text{ and } x \text{ ends with the letter y}\}$

$\mathbb{V} = \{x | x \in \mathbb{U} \text{ and } x \text{ begins with a vowel}\}$

$\mathbb{R} = \{x | x \in \mathbb{U} \text{ and } x \text{ ends with the letter r}\}$

a) Find $n\left(J'\right)$ b) Find $n\left(R \cap V\right)$

10.   I spent Spring Semester 2004 teaching at MJC's sister college up the hill in Columbia. A student there who was taking statistics did what I thought was a pretty interesting survey. She decided to survey other girls about what guys they would be willing to date based on what games the guys do (or do not) play.

Here are the findings:

Of 100 girls surveyed 75 said they would date guys who played chess, 11 said they would date guys who played Pokémon, and 6 girls said they would date guys who played Dungeons & Dragons. Twenty-two girls would not date guys who played any of these games. Three girls would date guys who played all three. Eight girls would date guys who played Pokémon and chess. Sixty-five of the girls said they would date guys who played only chess but neither of the other games.

Here are your questions:

   a) How many girls would date guys who played only Dungeons & Dragons?
   b) How many girls would date guys who played Pokémon and Dungeons & Dragons?

EXTRA CREDIT

Choose one of the following pairs of sets to compare in terms of cardinality. For the pair you have chosen determine if they have the same or different cardinality. State your result and support your answer fully with sound mathematical logic (as done in class).

OPTION 1: Compare $\mathbb{N}$ and $\mathbb{Z}$ 　　　　　 OPTION 2: Compare $\mathbb{N}$ and $\mathbb{R}$