Combo Session 5 & 6: Competition Problems & Games

"Sometimes I find mathematical problems difficult, but my old obsti-
nacy remains, for if I do not succeed today, I attack them again on the
morrow."

Mary Fairfax Somerville (1780-1872)

1.   A date is called a *weird* if the number of its month and the number of its day have
greatest common factor 1. What are the fewest number of *weird* days in a month?

A.   9   B.   10   C.   11   D.   14   E.   15

2.   If $AM/AT = .YC$ where each letter represents a different digit, AM/AT is in sim-
plest terms, and $A \neq 0$, then $AT =$

A.   15   B.   16   C.   25   D.   28   E.   75

3.   In the convex pentagon $AMTYC$, $\overline{CY} \perp \overline{YT}$, $\overline{MT} \perp \overline{YT}$, $YT = CY = 63$,
$MT = 79$, $AM = 39$, and $AC = 52$. Find the area in square units of the pentagon.

A.   5487   B.   5500   C.   5525   D.   5600   E.   5624

4.   A palindrome is a word or a number (like RADAR or 1221) which reads the same
forwards and backwards. If dates are written in the format MMDDYY, how many dates in
the 21st century are palindromes?

A.   1   B.   12   C.   24   D.   36   E.   144

5.   If each letter in the equation $\sqrt{AMATYC} = MYM$ represents a different decimal
digit, find T's value.

A.   3   B.   4   C.   5   D.   6   E.   7

TWENTY-ONE PLUS

Most or all of us probably remember some letter or number games from childhood. Perhaps you remember games from long car trips like finding all the letters of the alphabet on signs or license plates or games you might hear on the playground like "eenie-meenie-minie-moe" or "I one it, I two it . . . I jumped over it and you ate it!" One such counting game has the following rules:

> This is a two person game, and the winner is the person who says "21." We start with the number 1, and each of us can count one or two or three numbers at a time."

Play this game a number of times over the next few weeks (you might want to keep a record of which numbers each player says), think about strategy as you play, and then answer the following questions:

1. Is there a strategy that will allow you to win every time? If so, how?

2. Can you win this game in general even if you are counting to a number other than 21 and/or can count by groups of more or less than three numbers?

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Wythoff's Nim

The 'Twenty-one Plus' game I've had you exploring this semester is actually one of many versions of 'Nim.' It is an ancient game said to have originated in China under the name *tsyan-shidzi*, meaning 'choosing stones.' The following is a version created in the early twentieth century by Dutch mathematician W. A. Wythoff. As with 'Twenty-one Plus,' your task is to discover a winning strategy first for a specific set-up and then for a generalized version.

Specific Set-up: Make two piles of pennies, one pile of 10 coins and one pile of 7 coins. On your turn you may take as many coins as you want from one pile, or you may take coins from both piles, but if you take coins from both piles you must take the same amount from each one. You must take at least one coin on your turn. The player who takes the last penny wins.

Generalization: Make two piles of pennies, each pile containing an arbitrary number of pennies. The rules then follow as above.